Desmos Analysis of the Binary Star WDS 19471-1953

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Abstract Speckle interferometry observation of the Binary Star WDS 19471-1953 on the 1.5-meter telescope at Mt. Wilson Observatory on 2023.484 provided a position angle of 83.3° and a separation of 0.552". The published orbit appeared to include three observations that were off by 180°. Desmos analysis—with the three values corrected and two new speckle interferometry observations—produced three new apparent orbits that were all equally good fits to these observations. We expect that just one of the apparent orbits will fit subsequent analysis that includes the times of observations and produces orbital elements.

Introduction

This paper analyzes a binary star, WDS 19471-1953, discovered in 1873 by S. W. Burnham. Some 26 visual observations were made of this binary prior to a 1991 observation by Hipparcos (the European Space Agency's astrometric space telescope). Since then, there have been seven published speckle interferometry observations (listed in the Washington Double Star Catalog by the U.S. Naval Observatory), and one other speckle interferometry observation made in June 2023 at Mt. Wilson Observatory and reported in this paper for the first time.

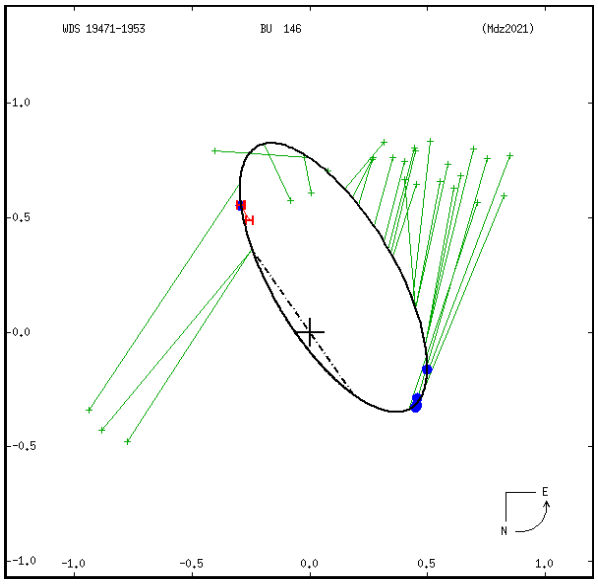

Figure 1: Orbital plot of WDS 19471-1953 from the Sixth Catalog of Orbits of Visual Binary Stars *(hereafter the 6th Orbit Catalog).*

Desmos (https://www.desmos.com/), an easily learned online graphing calculator, is used in geometry, algebra, trigonometry, and calculus classes in many high schools and colleges. It is free, online, and easily accessed. Results are stored in the cloud at no cost. Familiarity with Desmos allows student teams to efficiently tackle the analysis of binary star orbits. A three-step Desmos apparent orbit analysis process (Genet 2023) was used to analyze WDS 19471-1953. Each step is outlined below.

The objectives of this paper were to report the new speckle interferometry observation made at Mt. Wilson Observatory, derive an improved apparent orbit if that seemed appropriate, and evaluate the three-step Desmos analysis process.

Please see the Demos online graph at https://www.desmos.com/calculator/eani846wcj that was used to analyze this binary and produce the graphical images in this paper.

Step 1: Paste in and Adjust the Background Orbital Plot

As shown in Figure 2, the plot of WDS 19471-1953 that was downloaded from the 6th Orbit Catalog was pasted in as background in Desmos, rotated 90 degrees, scaled so that the 6th Orbit scale (tick marks) matched the Desmo scale, and translated so that the 6th Orbit primary star (0,0 + mark) matched the Desmos 0.0 origin. Please see Genet (2023) for details on this and other Desmos analysis procedures.

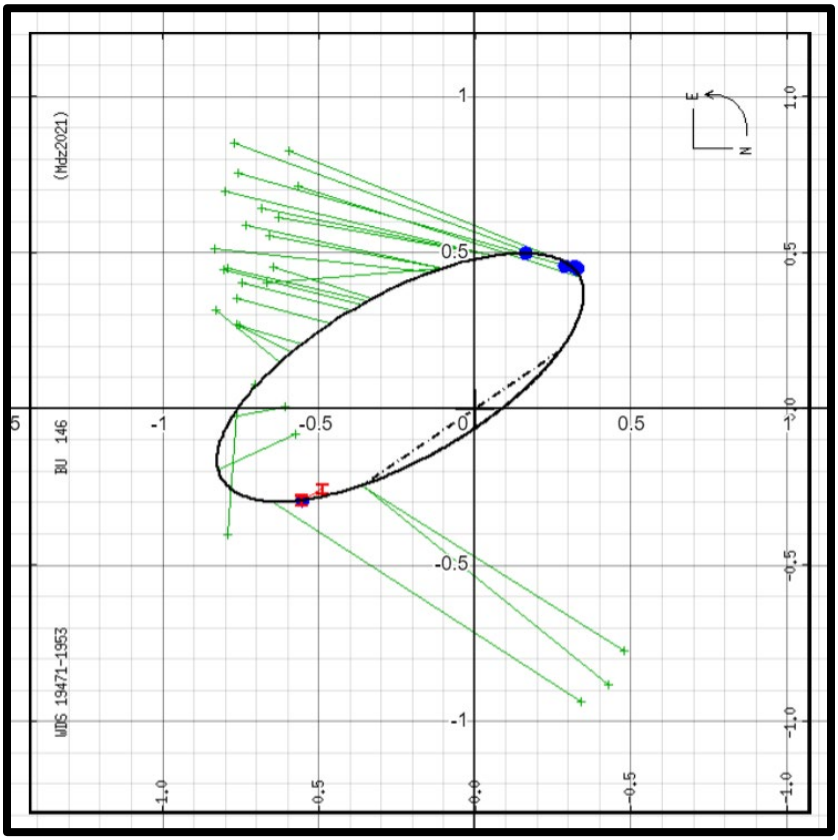

Figure 2: Background binary plot as it appeared in Desmos after appropriate rotation, scaling, and translation.

Step 2: Add New Observations and Label Previous Observations

The U.S. Naval Observatory's Washington Double Star Catalog is a database for all published observations of double stars (including binary stars). Information on the past published observations of any of the over 100,000 double stars in the catalog can be requested and will be supplied by return email as an attached text file. Space telescope and speckle interferometry observations received from the Naval Observatory for WDS 19471-1953 are shown in Table 1. The recent speckle interferometry observation made at Mt. Wilson Observatory (MWO) is also shown in the last row. Visual observations were not included as they were widely scattered (as can be seen from the plot) and the decision was made to only use the much more accurate space and speckle interferometry observations in the analysis.

Table 1: Key information on WDS 19471-1953. Column headings are the Label, Date (year and fraction of a year), position angle (PA) in degrees, the separation (Sep) in arcseconds, the aperture of the telescope (Ap) in meters, the Reference (code to the published observation), and the observational technique (Tech). The Tech codes are Hh Hipparcos, Ht Tyco, and the rest speckle interferometry (Sc CHARA 4m at CTIO, Su USNO, S ordinary speckle, and St Tokovinin HR Cam at SOAR).

Label	Date	PA	Sep	Ap	#	Referance	Tech
HIP1991	1991.250	27.7	0.6260	0.3	1	HIP1997a	Hh
Hrt1991	1991.720	28.1	0.6260	4.0	1	Hrt1996b	Sc
TYC1991	1991.750	27.6	0.5500	0.3	1	TYC2002	Ht
Tok2008	2008.539	53.8	0.5564	4.0	1	Tok2010	S
Tok2008	2008.765	54.2	0.5549	4.1	1	Tok2010	S
Tok2009	2009.262	55.1	0.5539	4.1	1	Tok2010	S
Mns2011	2010.591	57.8	0.5390	3.8	1	Msn2011d	Su
Tok2018	2018.251	72.0	0.5242	4.1	2	Tok2019c	St
Tok2019	2021.319	78.3	0.5130	4.1	2	Tok2022f	St
MWO2023	2023.484	83.3	0.5520	1.5	1	This paper	S

The observations in Table 1 were entered into Desmos, resulting in the graph shown in Figure 3. Note that the HIP1991, Hrt1991, and TYC1991 observational points on the right side of the plot were not associated with any points on the 6th Orbit background plot. The two red H symbols lying on their side (one over a blue dot for speckle interferometry) that appear in the lower left of the plot were not in agreement with the three 1991 observations listed in the Washington Double Star Catalog. Thomas Smith suggested that these points were in error by 180°, which was confirmed by plotting them as in Desmos with 180° added to their position angles. These points fell exactly on the lower left plot points, confirming the 180° error.

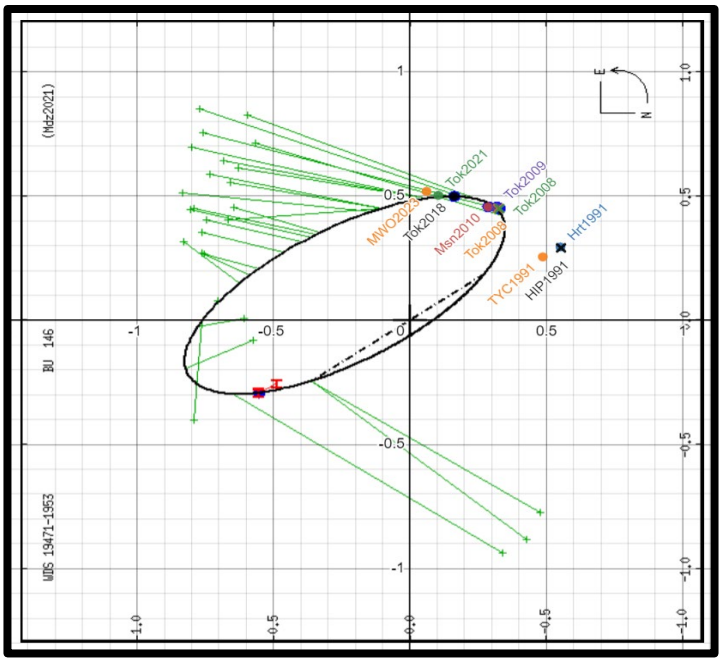

Figure 3: The Desmos graph for WDS 19471-1953 showing two new observations (MWO2023 and Tok2021, top middle) and eight previous space and speckle interferometry observations (now labeled).

Step 3: Create New Apparent Orbits

As can be seen from Figure 3, the published orbit of WDS 19471-1953 was not a good fit to the three corrected past observations (HIP 1991, Hrt1991, and Tyco1991, middle right). Desmos was used to generate a new apparent orbit to provide a better fit to the Hipparcos space telescope observation (the Tycho observation with its lower astrometric precision was not included in further analysis) and the speckle interferometry observations.

For an ellipse not centered on the origin and not coincident with the x-axis, the general parametric equations for an ellipse in the Cartesian plane are:

$x = (h + a \cos t)(\cos q) + (k + b \sin t)(-\sin q)$
$y = (h + a \cos t)(\sin q) + (k + b \sin t)(\cos q)$

where:
a is the semi-major axis
b is the semi-minor axis
h is the x offset of the center of the ellipse from the primary star (the "+")
k is the y offset of the center of the ellipse from the primary star (the "+")
q is the angle from the x axis to the ellipse major axis
(t is just the parametric variable that traces out the ellipse from 0 to 360°)

The values of the five parameters for a general ellipse were adjusted via Desmos sliders to obtain the fit shown in Figure 4. Two points, Tok 2021 and Msn 2010 were off slightly (around 10 mas). The ellipse parameters are shown to the left of the Desmos plot.

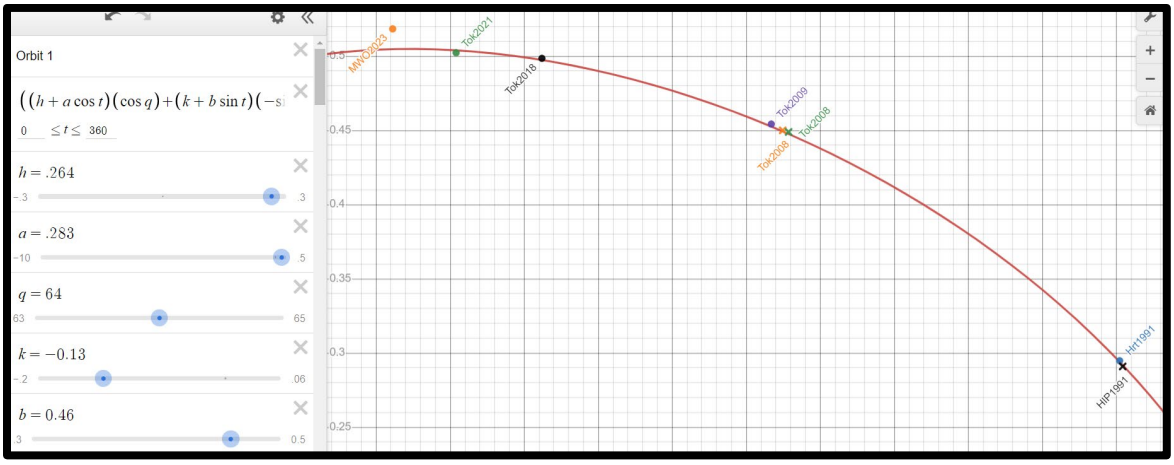

Figure 4: Desmos fit of an ellipse to eight high-quality observations.

As can be seen in Figure 5, the first new orbit is wildly different than the Mdz2001 published orbit. Assuming the three Mdz2002 points in the lower left (HIP 1991, Hrt1991, and Tyco1991) were in error by 180°, the new orbit appears to be a much better fit for the space telescope and speckle interferometry observations.

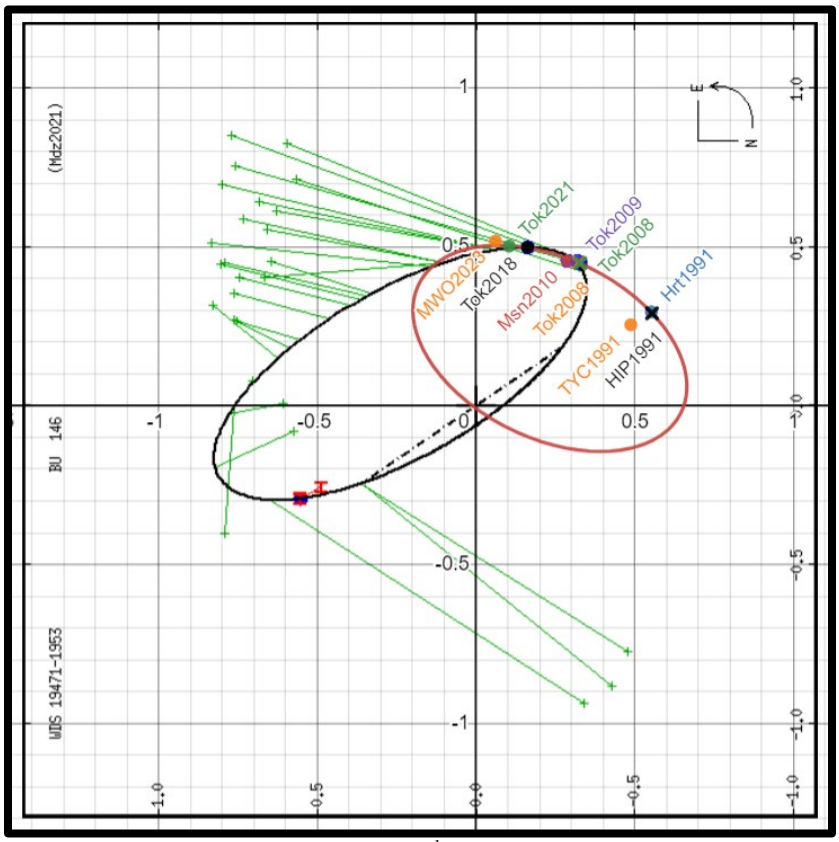

Figure 5: A comparison of the original orbit in the 6th Orbit Catalog (black) and the revised orbit (red).

Normally, only one new apparent orbit would provide a better fit to the observations than the original orbit. However, when the high-accuracy space and speckle interferometry observations only cover a small portion of the orbit, as is the case with WDS 19471-1953, more than one ellipse may make a reasonable fit to the observations. In the case of WDS 19471-1953, two additional apparent orbits were found that were also a good match to the space telescope and speckle interferometry observations as shown in Figures 6 and 7. The two observations, MWO2023 and Msn2010, still departed somewhat from the other observations.

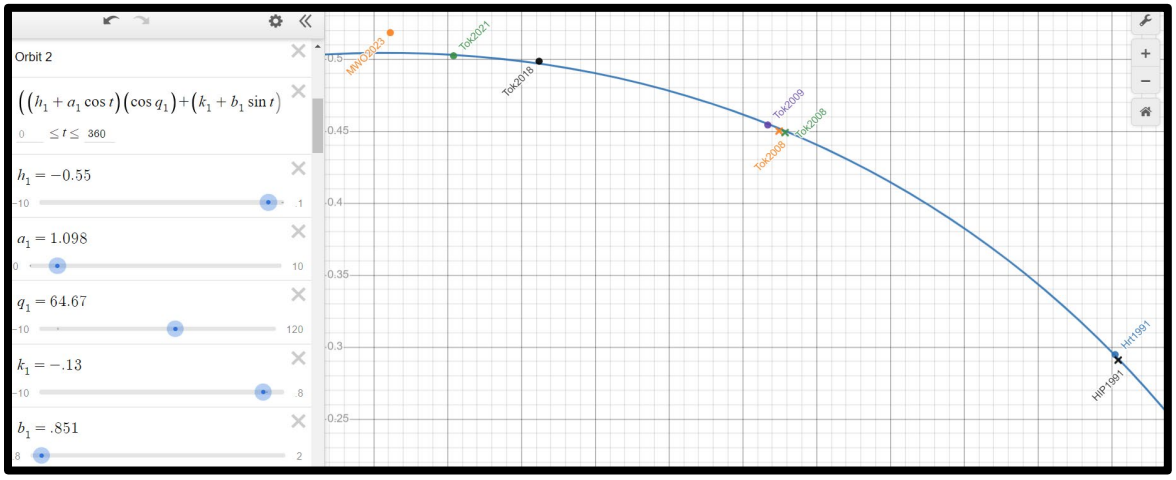

Figure 6: Desmos fit of an ellipse to nine high-quality observations.

Finally, a third new orbit was found that was also a good fit as shown in Figure 7

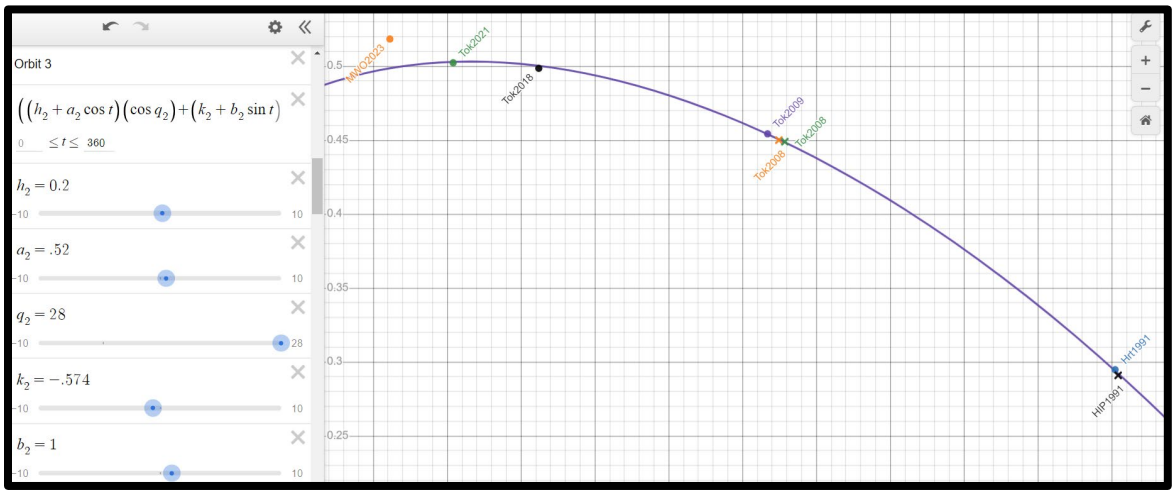

Figure 7: Desmos fit of an ellipse to nine high-quality observations.

The original 6th Orbit apparent background orbit (black), and all three of the new Desmos-derived apparent orbits are shown in Figure 8.

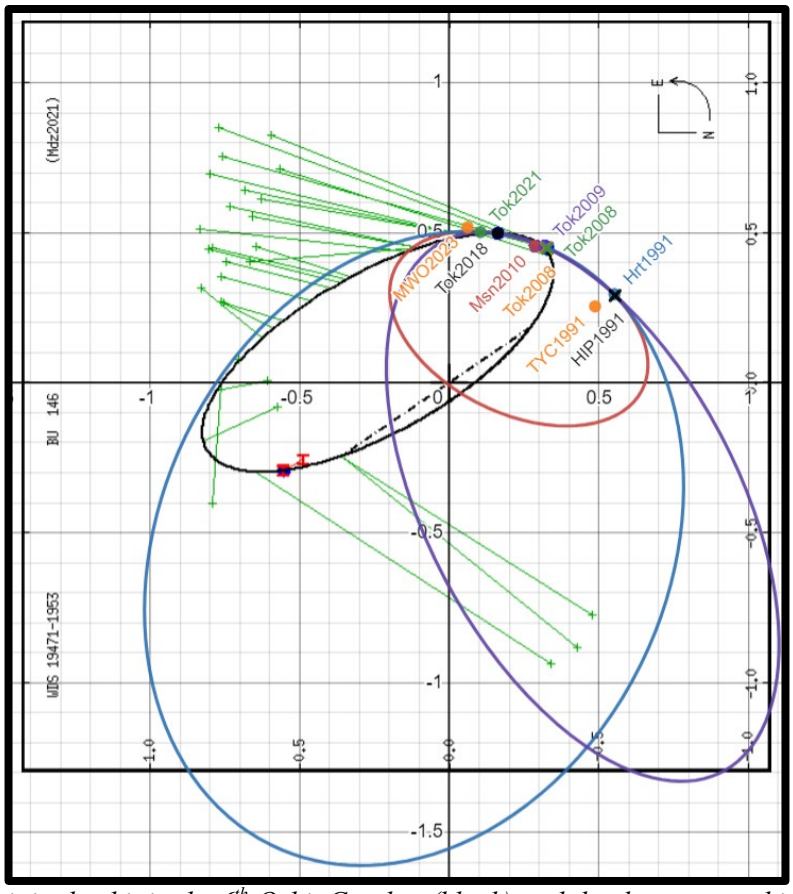

Figure 8: The original orbit in the 6th Orbit Catalog (black) and the three new orbits (red, blue, and purple).

The three new Desmos apparent plots shown above in Figure 8 are magnified to cover the portion of the orbit with the Hipparcos space telescope and speckle interferometry observations. As can be seen in Figure 9, all three match these observations closely except, as before MWO2023 and MSN2010. We expect that by including the date/time of each observation one of the three apparent orbits will be superior. These results will be included in a follow-up paper.

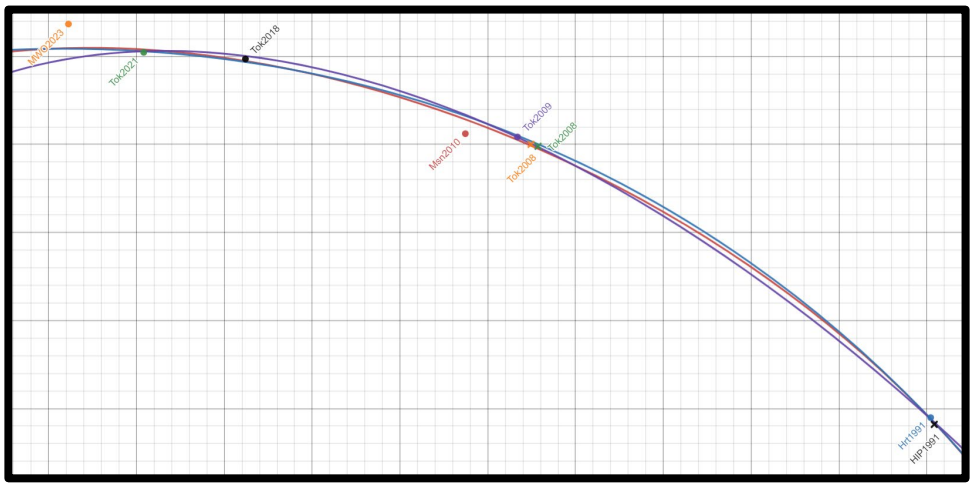

Figure 9: Desmos fit of the three new orbits (red, blue, and purple) to nine high-quality observations.

Conclusions

The new speckle interferometry observation of WDS 19471-1953 made at Mt. Wilson Observatory was reported for the first time in this paper. Three unexpected errors in the 6th Orbit plot of this binary were corrected.

While the published 6th Orbit plot was not a good fit for the corrected observations, using Desmos a much better fitting apparent orbit was found, although it was not unique as two additional equally good-fitting orbits were found. It is expected that further analysis that includes the date/time of the observations will determine which of these three orbits is the best fit.

The three-step Desmos analysis process worked well after adding several refinements. Desmos appears to be a useful tool for the analysis of binaries with apparent orbits plotted in the 6th Orbit Catalog.

Acknowledgments

Thanks to the United States Naval Observatory for the use of a plot from their *Sixth Catalog of Orbits of Visual Binary Stars,* and for supplying past observations from their *Washington Double Star Catalog*. Thanks also to Thomas Smith for his suggestion that the three points on the 6th Orbit plot that disagreed with the WDS values were all in error by exactly 180°. Gravic Inc. kindly provided travel funds for students to attend the run at Mt. Wilson Observatory. Rachel Freed, Reed and Chris Estrada, Nick Hardy, Leon Bewersdorff, Joseph Burch, Paul McCudden, Mark Harris and others either generated the target lists or managing the observations. We thank Thomas Meneghini, the Director of the Mt. Wilson Observatory, and his staff for their support during our observation run. Finally, we thank several external reviewers for their helpful comments.

References

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